French Didactique des Mathématiques and Lesson Study: a profitable dialogue?

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Abstract

Purpose – The purpose of this paper is to present French Didactique des Mathématiques (DM) to the Lesson Study (LS) community.

Design/methodology/approach – This theoretical paper presents the origins of DM in the Theory of Didactical Situations (TDS) by Brousseau. It elaborates about didactical engineering, fundamental situation and other fundamental concepts. It briefly presents other Didactique theories: the theory of conceptual fields, the anthropological theory of the didactic, the joint action theory in didactics and the double approach. It considers importance of the (TDS) and influences over teaching of mathematics. This paper finishes by highlighting the ways Didactique and LS could contribute to each other in a profitable dialogue.

Findings – The paper contrasts DM with some LS main features. It highlights the parallels despite fundamental differences in the initial goals of the perspectives. It shows that these differences could lead to productive dialogue by producing more practice-oriented forms of didactical engineering for the first and making teachers' principles for lessons more explicit for the latter.

Originality/value – The paper presents a very quick overview of the parallels between DM and LS. Additionally, this paper gives many accessible references in English for the reader to explore Didactique further.

Keywords Mathematics, Lesson Study, Anthropological theory of the didactic, Didactique, Theory of conceptual fields, Theory of didactical situations

Paper type Conceptual paper

Introduction

French *didactique des mathématiques* (DM) and Lesson Study (LS) have entirely different origins, theoretical background and research practice. Furthermore the two movements know quite little about each other. By coming from the first and discovering the latter, I was struck by the many common points and by the elements these two families of approaches could bring to each other. This paper is one part of a two-folded attempt to initiate a dialogue between these movements. One presentation, to be done in French to the DM community, will present the LSs in contrast with the DM development, following the footsteps of Miyakawa and Winsløw (2009a, b). Conversely, this paper presents DM to the LSs community by contrasting it with the main features of LS.

These parallels originated in our use of DM concepts for facilitating and analyzing a LS process in a research project currently conducted in Lausanne Laboratory Lesson Study: "From teachers' knowledge to classroom: a training and research setting." We will mention this LS in mathematics project as an illustration along this paper.

We will speak about LS (for e.g. Lewis and Tsuchida, 1997; Lewis and Hurd, 2011; Murata, 2011; Shimizu, 2014), but we do not exclude Learning Study (Pang and Marton, 2003; Runesson, 2014) and will mention the latter for specific aspects.

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Describing DM in English is a challenge in many aspects. The most obvious one is the question of translation of specific words. For that matter, I will rely on Warfield (2014) translations. The first word is *Didactique* itself. Since the adjective *didactic* has a pejorative sense of "overly inclined to lecture others," it has been difficult to promote its signification "as a noun denoting a field that studies questions raised by teaching and learning (of mathematical knowledge) in the milieu of school" (Warfield, 2014, p. v). Like Warfield, I shall use the French word *Didactique* or DM (for *Didactique des Mathématiques*) to avoid the pejorative connotation.

Obviously, presenting many aspects of a theory, moreover a family of theories, in one text will require some simplifications. The first is considering DM (and also LS) as a somehow unified movement. Nevertheless, we think these simplifications are necessary to give a flavor of these theories, but we hope some interested readers will look for the representative more developed text in English that we will mention. For this reason, we chose to give a lot of quotations and references in English, and, among them, many references to papers available online.

We will first present the origin of DM in the Theory of Didactical Situations (TDS) particularly in the work of Brousseau (1997). We will elaborate about didactical engineering methodology and fundamental situation concept. Other fundamental concepts will then be developed. We will then present briefly other DM theories: the theory of conceptual fields, the anthropological theory of the didactic (ATD), the joint action theory in didactics and the double approach. The following sections will consider the importance of TDS and its influences over the teaching of mathematics. The conclusion will sketch some of the ways DM and LS could contribute to each other in a profitable dialogue.

Brousseau and the TDS

It starts with a teacher, and with teachers

LS has its roots in classroom practice and in the desire to improve pupils learning. Roots in a teacher's work can also be seen in DM origins, back in the work of Brousseau, which we will present briefly.

As a primary school teacher in the 1950s, Guy Brousseau was fascinated with how children learn mathematics. He "was exposed to the work of Piaget, and greatly admired many of his ideas and explorations, but felt that Piaget's focus on individual children isolated from any group excluded some key learning dynamics" (Warfield, 2014, p. 1). Concomitant with the irruption of *Modern Math* into primary school teaching, Brousseau earned a math degree and worked with mathematician Lucienne Félix. Brousseau had a breath-taking ambition: "he wanted to re-design the entire French elementary mathematics education system" (Warfield, 2014, p. 3). A step toward this purpose was the redaction of a textbook. For that, he:

> […] worked with a collection of teachers from several schools in the area who were interested in experimenting with the ideas represented by the book. They made worksheets modeled on these ideas, gave them to their classes, made records of the results and gathered regularly to discuss them and produce more (Warfield, 2014, p. 4).

In the 1960's, the IREM[1] groups were created, in which faculty members from universities and schools could work together to do research on mathematics education. This structure, linked with Brousseau's grassroots work, led to the creation of the COREM[2]. This was a school with a completely unselected students' population for whom all the non-mathematical parts of the school day were conducted in a typical manner. Mathematics also typically happened in the regular classroom, but from time to time a class would be held in the observation center where it could be studied by teachers and researchers:

> Still firmly committed to the construction of a scientific theory, it expanded from the search for a scientifically defined mathematical program to the study of the conditions of the act of teaching. Didactique interested itself in everything to do with the acquisition of mathematical knowledge, including the knowledge about that acquisition which other fields contribute (Warfield, 2014, p. 5).

The main methodological tool in COREM was the didactic engineering (Artigue, 1994) which led to the development of many concepts. We now present this methodology with parallels to LS and some of the developed concepts.

From didactical engineering to fundamental situation

The COREM was in activity from 1973 to 1999. The experimentations:

> […] took place in the classroom, with analyses designed for the purpose and fully carried out. On the other hand, the objective of the analysis was not the improvement of a particular process or the smoothing of a particular lesson sequence, but rather the building of an entire theory that would serve as a foundation upon which to build didactical choices and decisions (Warfield, 2014, p. 6).

The goal of COREM was to build an entire theory, and to accomplish this the teachers and researchers conducted the particular qualitative methodology of didactical engineering on reiterated observations of the teaching.

Didactical engineering is "a form of didactical work that is comparable to the work of an engineer" (Artigue, 1994, p. 29). This methodological research tool consists first of an *a priori* analysis of the possible teaching of a mathematical subject: study of the teaching object as it already exists, analysis of constraints (of epistemological nature, cognitive nature, didactical nature), actual conception of the new teaching piece, making both global (macrodidactic) and local (micro-didactic) choices. This is followed by an *a posteriori* analysis based on iterations of the sequence of lessons by the group teachers. The final phase is an internal validation based on the comparison between the *a priori* analysis and the *a posteriori* analysis of the same situations (pp. 31-35).

On the one hand, the aim is to model teaching situations so that they can be developed and managed in a controlled way. On the other hand, the goal is the building of a theory. The accumulation of situations, for instance on the teaching of rationals and decimals (Brousseau *et al.*, 2014a) contributed to the building of the TDS:

> The theory of didactical situations, which is based on a constructivist approach, operates on the principle that knowledge is constructed through adaptation to an environment that, at least in part, appears problematic to the subject. It aims to become a theory for the control of teaching situations in their relationship with the production of mathematical knowledge (Artigue, 1994, p. 29).

The fundamental hypothesis of the TDS is that "every mathematical concept is the solution of at least one specific system of mathematical conditions, which itself can be interpreted by at least one situation" (Brousseau and Warfield, 2014, p. 165). Therefore, the knowledge can be modeled in a fundamental situation, or a family of situations, who will preserve and even give back the sense of this knowledge (Brousseau, 1998). This family of situations can be generated by changing the didactical variables of the fundamental situation. In this sense, the fundamental situation is generic as it "presents a model problem to which a large number of problems included in the learning objectives can refer" (Warfield, 2014, p. 52).

Three important parallels between TDS and LS can be stressed here. The first one is the primordial role of ordinary teaching practice in a somehow extraordinary setting in building inventive ways of teaching. The COREM setting and the research lesson setting are different, but they share this dialectic movement between ordinary and extraordinary teaching, between ordinary practice and research. The second feature is the link between the strong desire to better students' mathematical learning and the process of reform. Last, but not least, the third point is the essential role of a teaching setting (the lesson for LS, the fundamental situation for TDS) in being the actual realization, the model of a mathematical knowledge, therefore the possibility to work on the knowledge by working on the teaching setting. One could say that in both TDS and LS, the knowledge is seen as an object of teaching. Moreover, the essential elements of this knowledge are generated by the variation of the didactical variable in TDS and by the pattern of variation in Learning Studies. It seems to us that this generation is comparable. Comparing and contrasting (in the sense of Prediger *et al.*, 2008, pp. 171-172) the use of didactical variable in TDS and of pattern of variation in variation theory (Marton and Tsui, 2004) should be deepened, but this would go beyond the scope of this paper.

These parallels should not mask the differences between didactical engineering and LS, "both as regards their principles for "good" lessons, and as regards their objectives (Miyakawa and Winsløw, 2009a, p. 216)." Whereas LS:

> […] [is] oriented to develop and improve a lesson from the perspective of the people who participate in it, […] didactical engineering […] aims to establish scientific knowledge: the lesson is realized to confirm the conditions for learning which are anticipated in the a priori analysis of the target knowledge and previous experiment. In short, didactical engineering proposes a systemic approach to research on the conditions for learning mathematics, while lesson study proposes a systematic approach to developing mathematics teaching practice (p. 216).

These systemic vs systematic approaches within these parallels need to be considered when working on LS using TDS concepts, such as will be developed in the next section.

TDS developed many concepts that are widely used to analyze the teaching and learning of mathematics. We briefly describe some of the main concepts in the next section, including didactical contract, a-didactical situation, milieu, devolution, institutionalization, situation of action, of formulation, of validation and distinction between savoir and connaissance.

Some key concepts developed by TDS

The didactical contract is, primarily, an implicit "set of (specific) behaviors of the teacher which are expected by the student and the set of behaviors of the student which are expected by the teacher" (Brousseau *et al.*, 2014a, p. 204). These expectations have to be interpreted, outside the didactical situation, by the student, the teacher, the parents and the society. "The objective of these interpretations is to account for the actions and reactions of the partners in a didactical situation" (Brousseau *et al.*, 2014a, 2014b, p. 154).

Since the fundamental situation represents the knowledge, Brousseau's idea is to put the student in a position where they need this knowledge and could re-invent it. This part of the teaching situation, the didactic situation, where the student is answering directly to the problem, and not to the teacher, is called a-didactic:

> A Situation is a-didactical if the teacher's specific intentions are successfully hidden from the students and the student can function without the teacher's intervention. This doesn't mean that

> the students think the teacher is in the room simply as an entertainer, but that they are not conscious of the specific intent to have them learn some specific concept (Warfield, 2014, p. 12).

This complex process of getting the student to enter such a contract is called devolution:

> It is not a pedagogical device, because it depends in an essential way on the content. It consists of putting the student into a relationship with a milieu from which the teacher is able to exclude herself, at least partially (Brousseau and Warfield, 1999, p. 7).

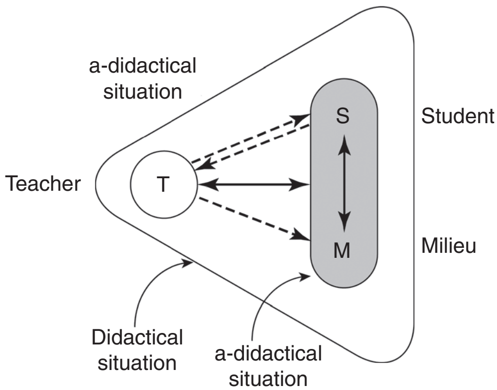
This milieu of a situation is defined by Brousseau as "what the students exercise their actions on and what gives them objective responses" (Brousseau and Warfield, 2014, p. 166). In fact, milieu is the usual translation for Brousseau's French term *milieu*, but, in French, it refers not only to the sociological milieu but it is also used in biology or in Piaget's work to describe the "environment." This interaction and the a-didactical situation can be represented as in Figure 1. Brousseau also proposed a model with a succession of interlocked milieus. This complex model, refined by Margolinas, cannot be presented here, but it is used to analyze complex classroom situations, *a posteriori* and *a priori* and to describe the presence of different and incompatible situations for the students and the teacher, as in (Margolinas *et al.*, 2005) or (Clivaz, under revison). We also use this structuration of the milieu to analyze lesson plans (*a priori*) and post lesson discussion (*a posteriori*) in our on-going LS in mathematics research.

The devolution, as a part of the didactical contract, presents both the teacher and the student with a paradoxical injunction. For the teacher:

> […] her aim is to lead the student to learn and understand some concept, and her indication that this concept has been learned will be some set of behaviors on the part of the student, but anything that she does that aims directly at producing that set of behaviors deprives the student of the opportunity to learn the concept itself.

And for the student:

> […] if he accepts that the contract requires the teacher to teach him everything, he doesn't establish anything for himself, so he doesn't learn any mathematics. On the other hand, if he refuses to accept any information from the teacher, the didactical relationship is broken and he can make no progress at all. In order to learn, he must accept the didactical relationship but consider it temporary and do his best to reject it (Warfield, 2014, p. 18).



Source: Based on Brousseau (1986, p. 88)

Figure 1.
The milieu of the teacher

But Brousseau does not consider this paradox as a contradiction. It reveals the tricky situation that the teacher will be often called to live in the classroom. It is a part of the teaching of mathematics and knowledge acquisition (Radford, 2008).

This paradox is at the heart of the TDS and it marks it fundamentally as constructivist approach. It also link TDS to teaching through problem solving approaches, since it:

> […] requires the teacher to provoke the expected adaptation in her students by a judicious choice of "problems" that she puts before them. These problems, chosen in such a way that the students can accept them, must make the students act, speak, think, and evolve by their own motivation (Brousseau, 1997, p. 30).

But this constructivism is not radical. If the devolution is maintained as long as possible, the students and the teacher engage in four phases of situations in TDS: action, formulation, validation and institutionalization. Students directly act (situation of action) on the milieu, articulate the ideas that have been developing implicitly (situation of formulation) and need to convince themselves or each other of the validity of the ideas they had developed (situation of validation). The final situation, institutionalization, marks the return of the teacher in the didactical situation. In this phase of the process:

> […] the teacher takes the ideas the class has developed, reviews them, shapes them up and if necessary provides them with labels. When it is pertinent, she provides the bridge between the class's production and the concepts and terms accepted by the world at large and in particular the standard curriculum (Warfield, 2014, p. 66).

So, for the TDS, "in opposition to constructivism, mathematical meanings and the mathematical forms of proving are not negotiable: they are part of the target knowledge, the cultural knowledge of reference" (Radford, 2008, p. 8).

This dialectical process, devolution and institutionalization, implicitly marked in the didactical contract, marks one primary source of the originality of the TDS:

> Classical teaching methods (internationally!) are based on institutionalization alone, without the creation of meaning: say what is to be learned, explain it and test for it. The researchers had been obsessed by a-didactical Situations because they are precisely what is traditionally totally lacking (Warfield, 2014, p. 13).

The modeling of the phasing of situations in TDS (action-formulation-validation-institutionalization) also echoed the structure for a typical lesson in Japan identified by Shimizu (1999), Stigler and Hiebert (1999) and cited by Miyakawa and Winsløw (2009a):

- Hatsumon (questioning): the teacher introduces an "open problem."
- Kikan-shido (instruction at the desk): the students work on the problem while the teacher circulates to observe their work and identifies different approaches, and clarifies the problem if needed.
- Takuto (orchestra conducting): the teacher asks students to successively present their solutions or ideas to the whole class.
- Neriage (elaboration): discussion of the validity and pertinence of the proposed ideas, mainly based on students' contribution.
- Matome (summing up): the teacher recalls the main points of the lesson, sometimes pointing out or even reformulating the best or new methods found.

A key distinction linked to this modeling of knowledge is the one between savoir and connaissance. This distinction and the dialectic movement paralleling the devolution-institutionalization is almost impossible to translate in English because of the single word in English, knowledge, to encompass the two in French. Nevertheless, some attempt (Warfield, 2014, pp. 68-69) and, when making the distinction, we will prefer to leave the terms un-translated. For Margolinas (2004, p. 44), a savoir in some institution is defined by a set of situated knowledge. Connaissance (in a situation) and savoir (in an institution) are linked by the processes of devolution and institutionalization (Figure 2).

An example of this movement is given by Brousseau and Warfield (2014): "For example, a theorem that the student knows very well (savoir), but about whose usefulness in a situation is unsure, functions provisionally as a simple piece of nonestablished knowledge (connaissance)" (p. 166).

To describe this complex movement and the realization of actual situations in ordinary classrooms, many concepts have been developed in DM, like the Topaze effect and the Jourdain effect (e.g. see Warfield, 2014, p. 70) or the object-tool dialectic (Douady, 1991).

The concepts described in this section are widely used by researchers, but also, in a sometimes simplified way, by textbooks authors and by teachers as described below.

Like Miyakawa and Winsløw (2009a) who used TDS framework to analyze a Japanese LS, our LS project in Mathematics uses TDS framework to analyze LS process. TDS will also be present in the process itself.

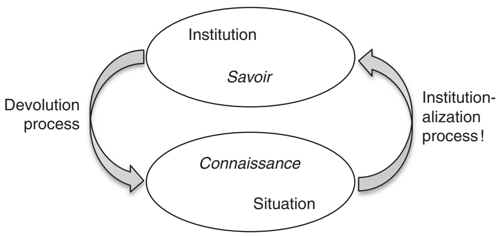
Some of these concepts have been more developed and gave birth to new and somehow independent theories, as we will see in the next section. We will describe two of them, the theory of conceptual fields and the ATD and name, in a shorter way, some other developments.

Other theories in DM

Two other theories were developed besides[3] the TSD by two important French researchers, Gérard Vergnaud and Yves Chevallard. The web site of the French Association for Research in Didactique des Mathématiques (ARDM) has a page in English about each of the three main character "founding father" of DM: Brousseau[4], Vergnaud[5] and Chevallard[6].

The theory of conceptual fields

Gérard Vergnaud (1998, 2009) developed the theory of conceptual fields as a developmental psychologist in a Piagetian perspective about the learning of mathematics. He is considered as a didactician as well as a psychologist. Like Brousseau his goal is mostly theoretical. The level of analysis is not a situation (micro-didactic) but a broader set around a concept. This level is qualified by Margolinas (2005) as "meso-didactic."



Source: Margolinas (2012, p. 8)

Figure 2. Connaissance and savoir

A conceptual field is constituted of:

- the tasks which give meaning to the concept (reference);
- the invariants on which the operationality of the schemes is based (content); and
- linguistic and non-linguistic forms which allow the concept, its properties, the situations and processing procedures to be represented symbolically (signifier).

Vergnaud's theoretical work about the conceptual field of addition is widely spread in teacher initial training in French speaking countries, and also in textbooks and in official curriculum. For example, the *Plan d'Etudes Romand* (CIIP, 2011) uses Vergnaud's classification to indicate which type of additive problem should be used in the French speaking part of Switzerland and official textbooks use this framework to analyze the additive problems. Therefore it is a tool used by teachers in preparing their lessons in primary grades, and it is a tool that could be used by LS groups working on additive problems, but also on multiplicative problems.

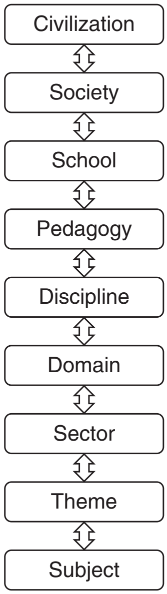
The ATD

Chevallard's first big contribution to DM was the concept of didactic transposition (Chevallard, 1985; Chevallard and Bosch, 2014). This concept gives a broader point of view to DM since it studies the conditions for a scholarly knowledge (*savoir savant*) to be transposed into a taught knowledge (*savoir enseigné*). This didactic transposition is a "social construction with multiple actors and different temporalities, through which some of these bodies of knowledge have to be selected, delimited, reorganised and, thus, redefined until reaching the classroom" (Bosch and Gascón, 2006, p. 55). This ecological point of view marks the development of the ATD, putting always the knowledge in relation with the institution and describing mathematical activity as an ordinary human activity. This is the case of the mathematical praxeology:

> A praxeology is, in some way, the basic unit into which one can analyse human action at large. […] What exactly is a praxeology? We can rely on etymology to guide us here – one can analyse any human doing into two main, interrelated components: praxis, i.e. the practical part, on the one hand, and logos, on the other hand. "Logos" is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning – particularly about the cosmos. […] [According to] one fundamental principle of ATD – the anthropological theory of the didactic – , no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. Praxis thus entails logos which in turn backs up praxis. For praxis needs support just because, in the long run, no human doing goes unquestioned. Of course, a praxeology may be a bad one, with its "praxis" part being made of an inefficient technique – "technique" is here the official word for a "way of doing" – , and its "logos" component consisting almost entirely of sheer nonsense – at least from the praxeologist's point of view! (Chevallard, 2006, cited by Bosch and Gascón, 2006, p. 59).

This point of view gets even broader with the levels of determination. These levels allow identifying conditions that go beyond the narrow space of the classroom and the subject that has to be studied in it (Figure 3).

ATD has been used to study LS process (for e.g. Elipane, 2012; Miyakawa and Winsløw, 2013; Winsløw, 2012) and the anthropological point of view seems particularly adequate to describe LSs in their institutional context.



Source: Bosch and Gascón (2006, p. 61)

Figure 3. Scale of levels of determination

Other theories

Other movements in DM were influenced by these three theories and mainly by TDS:

> […] most of the research [in Didactique since the sixties] has used the theory of didactical Situations, or positioned itself relative to it, or asked it questions or even contributed to its evolution (Perrin-Glorian, 1994, p. 97, cited by Warfield, 2014).

We will cite two of them. The first one is the "joint action theory in didactics" (Ligozat and Schubauer, 2010; Sensevy, 2012) which has emerged from the TSD and the ATD by focussing on the very nature of the communicational epistemic process within didactic transactions (Chevallard and Sensevy, 2014). This theory has been used in cooperative engineering which share with LS its iterative structure (Sensevy *et al.*, 2013), and it is used by many researches to describe classroom situations in many subjects (comparative didactics).

The second one is the double approach, a didactic and ergonomic approach for the analyses of teaching practices (Vandebrouck, 2013). With both it's didactique and ergonomic aspects, this theory allows analyzing professional development and teacher's practices in mathematics classrooms:

> The connection between the theory of activity and the "two constructivisms" thus offers a theoretical tool for a double approach from the viewpoints of mathematical didactics and the activity of the subjects in question (teacher and students). In particular, the Piagetian theory looks "from the student's side" at epistemological analyses of the mathematical objects in play, while the Vygotskian theory takes into account the didactic intervention of the teacher, mediating between knowledge and student in support of the student's activity (Rogalski, 2013, p. 20).

This connection between teachers' activity and mathematical didactics seems particularly adapted to analyze the work of the teachers in a LS process. On-going doctoral work about evolution of teachers practices during a LS process is using this framework (Batteau, 2013).

Importance of the TDS

Even if translated and promoted in English, (e.g. Brousseau, 1997; Brousseau *et al.*, 2001, 2004, 2007, 2008, 2009, 2014a, b; Brousseau and Gibel, 2005; Brousseau and Warfield, 2014; Brousseau and Warfield, 1999) particularly thanks to Virginia Warfield and by her mirror web site of Brousseau's web site (Brousseau, 2014), TDS is not really known in the English speaking math education world. "Nonetheless, despite a rich and growing literature base, it has as yet had only modest influence in the Anglophone world" (Warfield, 2014, p. v). However, Brousseau's TDS has been one of the most foundational and influential theory in didactique (not only in mathematics didactique) in France, as well as in Spain, Germany, Italy, Brazil, Chile and Mexico.

More importantly, "the detailed epistemic analyses of fundamental situations, their engineering and control in the classroom by the teacher, have helped mathematics educators understand the key role of suitable problems in the development of students' mathematical thinking" (Radford, 2008, p. 10). This influence was internationally recognized in 2003 when Guy Brousseau was awarded the first Felix Klein Medal of the International Commission on Mathematical Instruction[7]. The Hans Freudenthal Medal for 2009 was then obtained by Yves Chevallard[8] and 2013 Felix Klein Medal was awarded to Michèle Artigue[9], making French DM particularly recognized by ICMI. French speaking DM author are also numerous in the recent Encyclopedia of Mathematics Education (Lerman, 2014) and have been mentioned in this paper.

Influence on mathematics teaching

The IREM groups

In France, the link between academics and practising teachers in mathematics has been promoted in the IREM[10]. Those institutes, founded in the late 1960s accompanied the creation of DM, in Bordeaux with Brousseau, but also in Paris, Lyon or Strasbourg. These institutes regroup faculty members from universities and teachers from schools, so that they could work together and do research on mathematics education. Nowadays the 28 IREMs in France have as their mission to support the collaboration of practicing and research teachers (elementary to college level), and researchers with school administrators:

> If fundamental research plays an important role, IREM also has as a mission to simultaneously conduct a specific type of applied research and development about the teaching of mathematics. These researches aim to create a more efficient articulation between fundamental research, teaching and teacher training. The research at IREM is conducted in working groups[11] (IREM Paris 7, 2013, p. 13).

In Paris Diderot's IREM for instance, more than 150 teachers and academics participated in more than 20 thematic groups. Participants are compensated for their participation. Most IREM groups are generally assembled around a researcher (sometimes a PhD student). They meet on average monthly, and their main activity is to test and improve some theories in the classrooms of the participants. The groups publish their work in brochures which are now available online[12] and serve as a resource for other teachers, teacher trainers and researchers.

Compared to LS groups, IREM groups' initial impulsion and primary focus is on research. The initiative and the leadership is not in the teachers' hands. But, like LS groups, they also forge links between research and actual teaching, between researchers and teachers and they also spread their results for other teachers and are a medium for the dissemination of new mathematics teaching strategies.

DM, teacher training, textbooks and frameworks

The development of DM in the 1990s paralleled the transformation of initial teacher training in France. The creation of IUFM[13], replacing the normal schools, allowed many researchers in DM to get involved in initial teacher training. This exposed most teachers with DM concepts. This influence can be seen in books that were preparing students for competitive exams (e.g. Charnay and Mante, 2006). Many textbook collections also had DM researchers as members of editorial teams and were influenced by DM concepts (e.g. Chapiron *et al.*, 2001; Charnay *et al.*, 2007; ERMEL, 1978-2006; Peltier *et al.*, 2006). The 2002 and 2007 French national curriculum also were influenced by DM (Fagnant and Vlassis, 2010). In French speaking part of Switzerland, the *Plan d'Etudes Romand* (CIIP, 2011) and the mandatory mathematics textbooks have a preeminent DM influence:

> The creation of these new textbooks is based on choices that take into account the most recent knowledge about teaching. Two main scientific fields constitutes the reference: cognitive sciences […] and didactique which analyses the conditions in which school development and learning have the best chances to happen harmoniously[14] (Gagnebin *et al.*, 1998, p. 40).

But, as some researches in French speaking parts of Switzerland showed (Tièche Christinat and Delémont, 2005) and as our observations confirmed (Clivaz, 2012, 2014), this influence is not so present in everyday practices, and some important components of DM, present in teacher initial and continuous training, for example phases of institutionalization, are often lacking. This observation, compared with effects of LS seen in some countries, especially Japan, is a motivation to try new forms of teacher professional development, especially LS.

Conclusion

Our parallel exploration of DM and LS make us think, like Miyakawa and Winsløw (2009a), that this dialogue could bring some interesting developments to each other:

> Each approach takes into account different dimensions of mathematics education, and each of them could support the other, in order to develop a body of scientific knowledge in our domain on the one hand and to develop the practice of teaching mathematics in a specific culture on the other (p. 200).

More precisely, LS practice could lead DM to produce new and more practice-oriented forms of didactical engineering, in the direction pointed by Perrin-Glorian (2011) about "didactical engineering for development and training" also called "second generation didactical engineering." It could also give a fruitful kind of laboratory to observe actual teacher's practices, from planning to giving and reflecting about a lesson.

In the other direction, DM could be a tool to examine more explicitly teachers' principles for lessons, which are often implicit in LS, "as regards what aspects of mathematical knowledge are at stake and how different elements in the lesson design could affect students' learning. This could fertilize teachers' practice in lesson study" (Miyakawa and Winsløw, 2009a, p. 217). This impact on LS practice could also be a contribution to on-going efforts surrounding theoretical work on LS. By making content specific principles explicit, DM could contribute to improving the understanding of LS from a theoretical perspective, as called for by Potari (2011): "to compare lesson study to existing international theory and research on mathematics teacher education would improve our understanding of lesson study and provide us with frameworks to investigate further its role and importance (p. 132)."

Making research that is dialectically practice-oriented explicit about theories used and working toward theoretical developments is an objective we will pursue in our research in DM about the LS process in mathematics.

Notes

1. Institut de Recherche pour l'Enseignement des Mathématiques: Institute for Research on Mathematics Teaching.
2. Centre d'Observations et de Recherches sur l'Enseignement des Mathématiques: Center for Observation and Research on Mathematics Teaching.
3. For more information about the complex links and theoretical differences between these theories, please see Margolinas (2005).
4. www.ardm.eu/contenu/guy-brousseau-english, accessed January 3, 2015.
5. www.ardm.eu/contenu/gérard-vergnaud-english, accessed January 3, 2015.
6. www.ardm.eu/contenu/yves-chevallard-english, accessed January 3, 2015.
7. www.mathunion.org/icmi/activities/awards/past-recipients/the-felix-klein-medal-for-2003/, accessed December 10, 2014.
8. www.mathunion.org/icmi/activities/awards/past-recipients/the-hans-freudenthal-medal-for-2009/, accessed December 10, 2014.
9. www.mathunion.org/icmi/activities/awards/the-felix-klein-medal-for-2013/?no_cache=1&sword_list%5B%5D=klein, accessed December 10, 2014.
10. Institut de Recherche sur l'Enseignement des Mathématiques: Institutes for Research on Mathematics Teaching.
11. My translation.
12. see www.irem.univ-paris-diderot.fr/sections/publications/ for Paris IREM publications, accessed December 10, 2014 or http://publimath.irem.univ-mrs.fr, accessed December 10, 2014 for the database of all publications.
13. Institut Universitaire de Formation des Maîtres, University Institute for Teachers Training.
14. My translation.

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